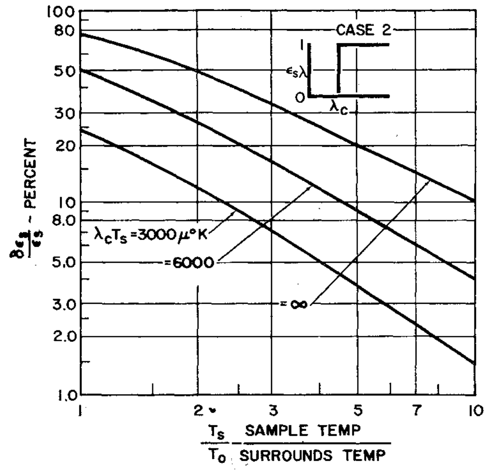

Fig. 2 Nongray error for selective emitter-type material.

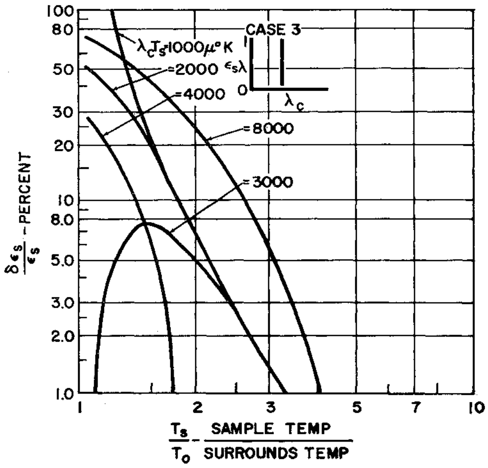

Fig. 3 Nongray error for spike emitter-type material.

"spike" or single emittance line. Edwards and Nelson[1] presented partial results for case 1.

Making the substitutions

$$F(Z) = C_1 Z^{-5}/\{[\exp(C_2/Z)] - 1\} = T^{-5}E(\lambda,T) \quad (10a)$$

$$Z = \lambda T \quad (10b)$$

$$T^* = T_s/T_0 \quad (10c)$$

in Eq. (8) and applying the limits from Eq. (9), the nongray error for the cases considered becomes

Case 1

$$\frac{\delta\epsilon_s}{\epsilon_s} \geq \frac{1}{[(T^*)^4 - 1]} \left[\int_{\lambda_c T_0}^{\lambda_c T_0 T^*} F(Z)dZ \Big/ \int_0^{\lambda_c T_0 T^*} F(Z)dZ\right] \quad (11a)$$

Case 2

$$\frac{\delta\epsilon_s}{\epsilon_s} \geq \frac{1}{[(T^*)^4 - 1]} \left[\int_{\lambda_c T_0}^{\lambda_c T_0 T^*} F(Z)dZ \Big/ \int_{\lambda_c T_0 T^*}^{\infty} F(Z)dZ\right] \quad (11b)$$

Case 3

$$\frac{\delta\epsilon_s}{\epsilon_s} \geq \frac{1}{[(T^*)^4 - 1]} \frac{F(\lambda_c T_0) - T^*F(\lambda_c, T_0, T^*)}{T^*F(\lambda_c, T_0, T^*)} \quad (11c)$$

The results of numerical computation of Eq. (11) are shown in Figs. 1–3 for a range of T_s^* and $\lambda_c T_s$. If $\lambda_c T_s$ is equal to 3000, the material has an emittance of 0.27 if it is a case 1 material and 0.73 if it is a case 2 material; if the value of $\lambda_c T_s$ is 6000, the corresponding emittances are 0.73 and 0.27, respectively. The extremes of $\lambda_c T_s$ cannot be ignored, however. For both case 1 and 2 materials, the extremes of $\lambda_c T_s$ are highly reflective materials, and these extremes indicate the possible effect of a surface film that is transparent through almost all of the wavelengths involved in the emission. The nongray error can be minimized by maintaining a large value of T_s^* in the instance of case 1 or 3. The error for case 2 is significant for values of T_s^* as large as 5–10.

Reference

[1] Edwards, D. K. and Nelson, K. E., "Maximum error in total emissivity measurements due to nongrayness of samples," ARS J. **31**, 1021–1022 (1961).

Erratum: "Electrical Discharge Across a Supersonic Jet of Plasma in Transverse Magnetic Field"

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[AIAA J. 1, 234 (1963)]

LINE 7 of the above paper should read "... at a velocity of approximately 3×10^3," rather than "... 3×10^4." The value was correct in galley proof, but the error occurred in making another correction in this paragraph in page proof.

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Moment of Momentum by Direction Cosines

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THE equation relating the sum of the moments acting on a rigid body to the time rate of change of its moment of momentum can be written

$$\mathbf{M} = [C_{01}][\dot{C}_{10}][I]_1([C_{01}][\dot{C}_{10}]) + [I]_1([\dot{C}_{01}][\dot{C}_{10}] + [C_{01}][\ddot{C}_{10}]) + m\,\mathbf{r}_1 \times \mathbf{a} \quad (1)$$

where $X_0Y_0Z_0$ is an inertial Cartesian coordinate system and $X_1Y_1Z_1$ is *any* coordinate system fixed in the body. Its origin is 0_1.

$[C_{01}]$ is the direction cosine matrix taken from the transformation matrix that transforms the coordinates of a point *from* system 0 *to* system 1. $[C_{10}] = [C_{01}]'$, the transpose of $[C_{01}]$. $[I]_1$ is the inertia tensor of the body computed in system 1. $\mathbf{r}_1$ is the position vector of the center of mass of the body measured from 0_1, and $\mathbf{a}$ is the acceleration of 0_1 relative to $X_0Y_0Z_0$.

$\mathbf{M}$ is the sum of the moments, about 0_1, of all the external forces acting on the body. Equation (1) gives its components in system 1.

The curved brackets denote the operation of forming (either manually or by computer program), a column matrix

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from certain elements of the square matrix enclosed by the brackets, viz., the elements (3, 2), (1, 3), (2, 1), and in that order in the column, reading downward [see Eq. (3)].

It will be seen that (1) is the classical equation

$$\mathbf{M} = \boldsymbol{\omega}_{01} \times \mathbf{H} + (\dot{\mathbf{H}})_{X_1Y_1Z_1} + m\,\mathbf{r} \times \mathbf{a} \qquad (2)$$

from the fact that

$$[\Omega_{01}]_1 \triangleq \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}_{01\cdot 1} = [C_{01}][\dot{C}_{10}] \qquad (3)$$

This is the angular velocity matrix of system 1 relative to system 0, expressed in system 1 components.[1] It may be transformed to system 2 by a similarity transformation.

$[I]$ may be transformed from *any* other system, say system 2, to system 1 by

$$[I]_1 = [C_{21}]\{[I]_2 + m\{[r_2][r_2] - [r_{21} - r_2][r_{21} - r_2]\}\}[C_{12}] \qquad (4)$$

where $\mathbf{r}_2$ is the position vector of the center of mass of the body in system 2, and $\mathbf{r}_{21}$ is the position vector of 0_1 in system 2. The square matrices are formed from the vectors as in Eq. (3).

The coordinates of the origins 0_1 and 0_2 are related by

$$(0_1)_2 = -[C_{12}](0_2)_1 \qquad (5)$$

Note that the velocity, of each element of mass of the body, used in calculating the moment of momentum was measured relative to a coordinate system with origin 0_1 but not rotating relative to inertial space. This system, in general, is noninertial because of **a**.

A direction cosine matrix is highly redundant. Only four elements, not in the same minor, and no three in the same row or column need be specified. The other elements are functions of these, for example, each element equals its cofactor. Considerable time may be saved when multiplying matrices by computing only those elements that are to be used, as in Eq. (3).

Reference

[1] Lur'e, A. I., *Analytical Mechanics* (Gosudarstvennoe Izdatel'stvo Fiziko-Matematicheskoy Literatury, Moscow, 1961), Eq. (2.12.5).

Comment on "A Statistical Optimizing Navigation Procedure for Space Flight"

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IN Battin's recent paper,[1] he refers to a method whereby an optimum linear estimator is formulated as a recursion operation in which a current best estimate is combined with newly acquired information to produce a still better estimate. Battin states that the original formulation of this method was presented by Kalman and the original application to space navigation was made by Schmidt and his associates.

The author would like to call attention to his following publications, both antedating those of Kalman and Schmidt, in which this recursive estimation method, generalizations thereof, and its application to trajectory estimation, are described fully.[2, 3] This work was done in late 1957.

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References

[1] Battin, R. H., "A statistical optimizing navigation procedure for space flight," ARS J. **32**, 1681–1696 (1962).
[2] Swerling, P., "A proposed stagewise differential correction procedure for satellite tracking and prediction," Rand Corp. Paper P-1292 (January 8, 1958).
[3] Swerling, P., "First order error propagation in a stagewise smoothing procedure for satellite observations," J. Astronaut. Sci. **6**, 46–52 (Autumn 1959); also Rand Paper P-1674 (February 19, 1959).

Generalization of the Note "An Error Analysis in the Digital Computation of the Autocorrelation Function"

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WITH regard to my recent paper,[1] it was brought to my attention by W. J. Stronge and D. L. Smith that not only does Eq. (5) appear to be in error, but also inequality (10) would appear to be valid if, and only if, $F(t)$ is a monotonic increasing function of t on $-T \leq t \leq T - \tau$. These comments will be discussed in this note.

Equation (5) is in error and should read

$$\Delta t = (2T - \tau)/(K - m) \qquad (1)$$

or

$$\Delta t = 2T/K \qquad (2)$$

It also should be noted from Eqs. (1) and (2) that the following assumption has been made:

$$\Delta t = \Delta\tau = 2T/K \qquad (3)$$

Hence,

$$\tau = m(\Delta\tau) = m(2T/K) \qquad (4)$$

This is the same as Eq. (4) in the forementioned paper.

Define

$$F(t) = f_1(t)f_1(t + \tau) \qquad (5)$$

and assume that $F(t)$ looks like the curve in Fig. 1. For ease in the analysis that follows, define a new function $G(t)$ as follows:

$$G(t) = F(t) + a \qquad a > 0 \quad (6)$$

where $G(t) \geq 0$ since $F(t) \geq -a$ for all t on $-T \leq t \leq +T$. It is evident from Eq. (6) that

$$\frac{1}{2T - \tau}\int_{-T}^{T-\tau} G(t)dt = \frac{1}{2T - \tau}\int_{-T}^{T-\tau} F(t)dt + a \qquad (7)$$

In order to consider the digital evaluation of the left member of Eq. (7), divide the interval $-T \leq t \leq T - \tau$ into $(K - m)$ equispaced subintervals, each of length $\Delta t_i = t_{i+1} - t_i$. Define A_i to be the area bounded by the curve $G = G(t)$, the t axis, between the lines $t = t_i$ and $t = t_{i+1}$, as shown in Fig. 2. Denote the maximum and minimum values of $G(t)$ in the interval Δt_i by U_i and L_i, respectively, and con-

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